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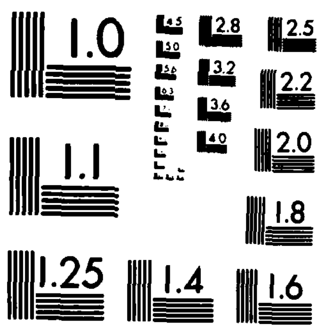
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Research during this period continued in the following topic areas: (1) Nonlinear filtering; (2) Large deviations problems; (3) Stochastic control of piecewise-deterministic processes; (4) Stochastic variational calculus; (5) Approximating multiple Ito integrals with band-limited processes; (6) Large deviations methods and asymptotic properties of recursive algorithms; and (7) Approximation and modelling for distributed stochastic systems. Progress is summarized and a list of publications supported by the grant is included in the report.

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Numerical Methods and Approximation and Modelling Problems in Stochastic Control Theory

Wendell H. Fleming
Harold J. Kushner

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Fleming and his associates conducted research on the following topics:

a) <u>Nonlinear filtering</u>. Analytical and numerical techniques for approximately optimal nonlinear filters were studied by Fleming's Ph.D. student R. McGwier. The analytical results take the form of a regular perturbation expansion, while the numerical results are based on moving finite elements. A preliminary report was given in Reference [1] with details in [2].

b) <u>Large deviations problems</u>. In [3] Fleming and Tsai obtained asymptotic estimates for minimum exit probabilities, associated with nearly deterministic controlled diffusion processes. The method used probabilistic and differential games ideas. Recently Fleming and Souganidis found a much simpler approach to those results, based on the viscosity solution technique in partial differential equations. These results were reported in [4], with a detailed paper in presentation.

Reference [5] explains a stochastic control approach to large deviations problems. Two kinds of large deviations problems are considered. One is for nearly deterministic Markov processes (including results of Ventsel-Freidlin type). The other is for Donsker-Varadhan type problems, including stochastic formulas for the dominant eigenvalue associated with a Markov process.

In reference [6] Sheu obtained large deviations results about the invariant density for nearly deterministic diffusions. These may be regarded as a first step toward the difficult problem of justifying a WKB-type expansion for invariant densities, including optimal results about regions where such expansions are possible.

c) Stochastic control of piecewise-deterministic processes. A wide class of control models are described in terms of dynamics which are deterministic between random times when jumps in the state of the system being controlled occur. Examples are resource exploration-consumption, storage, and virtual waiting time problems. In many instances there are in addition state space constraints. Fleming's Ph.D. student H. M. Soner has undertaken a study of such problems, in a series of papers [7], [8], [9].

d) Stochastic varaitional calculus. A stochastic variational calculus is described in [10]. When applied to a special class of fixed endpoint stochastic variational problems, it gives a new function space integral representation for fundamental solutions to linear second order parabolic partial differential equations.

Kushner worked on the following topics:

a) Approximating multiple Itô integrals with band-limited processes. Such problems arise in many applications: in using or approximating liklihood ratios under realistic noise conditions, in approximating non-linear filters by a sum of multiple integrals with respect to the observation, etc.

Let $n^{\varepsilon}(\cdot)$ denote a "wide-band width" vector valued process. We are concerned with limits of $\int_0^t\int_0^\tau n^{\varepsilon'}(\tau)L(\tau,s)n^{\varepsilon}(s)d\tau ds$, and for the m-multiple integral case. For the most important case, a weak convergence result is obtained, and the "correction" terms exhibited. The method is such that the conditions used can readily be weakened. An application to a likelihood functional and hypothesis testing problem is given. There, the weak convergence result (rather than mere convergence of finite dimensional distributions) is essential if the limit approximation is to make sense as an approximation to the likelihood functional. The correction terms depend only on the limit (as $\varepsilon \to 0$) of the correlation function of the (renormalized) $n^{\varepsilon}(\cdot)$. The conditions are reasonable with respect to the applications. See [11].

b) Large deviations methods and asymptotic properties of recursive algorithms. Recursive algorithms appear in numerous contents in control and communication. Typically, one is interested in quantities relating to error or to the escape time or probability of the estimate from some stable point of the algorithm (as in tracking problems where the error is the tracking error and the stable point is the origin). The large deviations methods provide a powerful tool for this.

Asymptotic properties of Robbins-Munro and Kiefer-Wolfowitz type stochastic approximation algorithms are obtained via the theory of large deviations. The conditions are weak and can even yield w.p.1. convergence results. The probability of escape of the iterates from a neighborhood of a stable point of the algorithm is estimated and shown to be considerably smaller than suggested by the classical 'asymptotic normality of local normalized errors' method of getting the asymptotic properties. The escape probaiblities are a natural quantity of interest. In many applications, they are more useful than the 'local normalized mean square errors'. Other large deviations estimates are also obtained. Typically, if $a_n = 1/n^\rho$, $\rho \leq 1$, then the probability of escape from a neighborhood of a stable point in some (normalized) time interval $[n,m]: \sum_n^m a_i \sim T$ is $\exp - n^\rho V_\rho$, where V_ρ does not depend on ρ for $\rho < 1$ and is the solution to an optimal control problem. If the noise is Gaussian, then the optimal control problem is relatively easy. Under quite broad conditions, inthe Kiefer-Wolfowitz case the control problem has the Gaussian form, whether or not the noise is Gaussian. The techniques are expected to be quite useful in the analysis of the asymptotic properties of recursive algorithms generally. See [12].

c) Approximation and modelling for distributed stochastic systems.

Noise driven evolution equations are a natural and commonly used model for 'distributed' stochastic systems. Typically one assumes 'white' noise, although in applications this is not reasonable. In two papers [13,14], summarized below, we show how to approximate the 'real' system by the 'ideal' system, in the sense of finding the best approximation, when the noise bandwidth is large.

Let B_2^ε (a first order differential operator), B_1^ε and B_0^ε be functions of x and of a 'wide-band width' random process ('band-width' as $\varepsilon \to 0$). Define u^ε by (A is an elliptic operator)

$$\frac{\partial u^\varepsilon}{\partial t} = Au^\varepsilon + \frac{1}{\varepsilon}[B_2^\varepsilon + B_1^\varepsilon]u^\varepsilon + \frac{1}{\varepsilon} B_0^\varepsilon, \quad u^\varepsilon(0) = u_0.$$

Under appropriate conditions, $\{u^\varepsilon(\cdot)\}$ converges weakly to a process $u(\cdot)$ which solves a stochastic PDE, driven by a cylindrical Wiener process. The treatment is entirely probabilistic. Methods which the authors applied previously to the finite dimensional cases are modified and extended for the current class of problems.

For an alternative approach consider: For a parabolic equation with wide bandwidth coefficients, we show that the solution converges weakly to that of a stochastic PDE driven by an infinite dimensional Wiener process as the bandwidth tends to infinity. The treatment is novel and purely probabilistic. The solution to the "wide band" coefficient system is represented as a conditional expectation of a functional of a certain diffusion. By a weak convergence argument, the conditional expectation (and its mean square derivatives) converges weakly to a conditional expectation of a functional of a "limit" diffusion. It is then shown that this "limit" functional satisfies the appropriate stochastic PDE. The infinite dimensional Wiener process is represented explicitly in terms of the original system noise. No coercivity or strict ellipticity conditions are required. The result provides a partial justification for the use of infinite dimensional Wiener processes in distributed systems. Since the method is based on weak convergence arguments for Itô-type equations with wide bandwidth coefficients and "PDE methods" are avoided, it is likely that the technique will find greater use in the analysis of infinite dimensional stochastic systems.

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